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ORIGINALNI NAUČNI RAD / ORIGINAL SCIENTIFIC PAPER

FRACTALITY OF CROATIAN AND SERBIAN STOCK MARKETS

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Abstract: *Empirical literature explaining stock markets behaviour develops in a several directions. One strand of literature supports efficient market hypothesis while other strand of literature suggests fractal market hypothesis. Existing literature presents mixed findings on long memory property and hypothesis of stock market fractality. This paper aims to contribute to the debate and examine multifractality and long memory property of returns in Croatian and Serbian capital markets while considering the role of trading columns. Using multifractal Detrended Fluctuation Analysis and daily returns from the beginning of 2010 up to the end of 2021 for CROBEX and BELEX15 empirical finding suggested multifractality and long memory or persistence in CROBEX and BELEX15 returns. Dynamics in trading volumes exhibited multifractality but no long memory property in case of CROBEX as well as in case of BELEX15. Price-volume cross-correlation in case of CROBEX as well as in case of BELEX15 can be described as mean reverting process with no long memory property.*

Keywords: *CROBEX, BELEX15, long memory, multifractality.*

JEL classification: *G14.*

INTRODUCTION

Returns on capital markets is an ever-green topic in empirical finance literature. One strand of literature considers efficient or adaptive market hypothesis and brings empirical results regarding validity of the hypothesis (Bosnjak, 2023) and literature herein. İcan & Çelik (2023) suggested that the markets in countries with well-functioning and more transparent public sector exhibits weak-form market efficiency. Other strands of literature put an effort to establish link from behavioural sciences to finance (Aslam et

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al., 2021) suggesting an irrational behaviour of entities on the market. Liu et al. (2022) provided differences and similarities between fractal market hypothesis and efficient market hypothesis. Croatia is one of the post-transition economies that recently joined the European Union. In the last few decades Croatia has experienced transformation of the economy emphasizing promotion of the free market structures. This transformation was accompanied by privatization and development of the financial system. With rising efficiency of the financial intermediaries the stock market has experienced significant growth and relationship between the stock market and the fundamental economic variables has become an important issue for scholars and practitioners (Novak, 2021).

This paper aims to consider persistence in returns and trading volumes on Croatian and Serbian stock market using multifractal detrended fluctuation analysis.

Besides introductory part, the remainder of this paper is organized as follows: Section 2 briefly summarizes existing literature related to the topic under consideration. Section 3 presents research methodology, while Section 4 empirical results and discussion. The final section provides an overview of the main findings of the research.

BRIEF LITERATURE OVERVIEW

The papers present mixed findings on the hypothesis of stock market fractality. Jiang and Zhou (2008) find that the so-called multifractality in intraday stock market indexes is merely an illusion. However, Zhuang et al. (2004) find that the rate of returns of the Chinese stock market does not obey the normal distribution, and that the stock price indexes exhibit fractal time series. Dar et al. (2017) finds that stock markets around the globe indicate the dominance of higher frequencies during crises periods, which validates the assertions of Fractal Market Hypothesis. Based on daily data between January 12, 2018 and January 12, Karaömer (2022) found long memory in returns supporting validity of fractal market hypothesis in Indonesia, Mexico, Turkey and Nigeria. Nargunam & Lahiri (2022) employed detrended fluctuation analysis the and provided empirical results against validity of market efficiency in Indian stock returns. Arashi & Rounaghi (2022) provided literature review for fractal market hypothesis and efficient market hypothesis along with analysis of NASDAQ stock exchange. The empirical finings suggested non-fractality but efficiency of NASDAQ stock exchange market. Lahmiri et al. (2022) pointed out fractal market hypothesis as the most appropriate approach to explain dynamics of returns on European equity markets. Novak (2019) based on quantile regression approach rejected weak form of the efficient market hypothesis for the Croatian capital market. Bošnjak (2023) employed variance ratio test and fix-length rolling window on daily CROBEX returns between September 1997 and July 2021 to examine market efficiency of Croatian stock market. The empirical findings were in favour of adaptive market hypothesis for Croatian stock market. The periods lower prices and higher liquidity were found as more likely to be inefficient. Krsikapa-Rasajski & Rankov (2016) evaluated efficient market hypothesis for Belgrade Stock Exchange BELEX 15 and BELEX LINE on daily data up to 31 December 2014. Empirical findings suggested that capital market in Serbia could not be considered sufficiently efficient. Schabek et al. (2019) based on data between September 2017 and March 2018 suggested semi-strong form of market efficiency. Overall, the papers suggest that the evidence for the hypothesis of stock market fractality is mixed and requires further investigation.

METHODOLOGY

This paper used multifractal Detrended Fluctuation Analysis (DFA) as a suitable approach to capture the structure of a series and determine the properties that define change of the series structure over time.

Method employed in this paper developed by Peng et al. (1995), Kantelhardt et al. (2002) and Jiang et al. (2019) involves detrended random walk like time series. Suppose random walk like time series $X(i)$ where $i = 1, \cdots N.$ Detrended residuals can be calculated following equation (1):

$$\in (i) = X(i) - \hat{X} \tag{1}$$

Where $\hat{X}$ denotes local polynomial linear trend function. The residuals $\in (i)$are devided into $N_s = int[N/s]$ segments of equal size such that segments do not overlap. So, v^{th} segment can be presented as $S = \in \left((v-1)s + j\right)$ while $j = 1, \cdots s.$ and fluctuation function or root mean square of detrended residuals can be calculated as given in equation (2):

$$[F_v(s)]^2 = \frac{1}{s}\sum_{j=1}^{s}\left[\in \left((v-1)s + j\right)\right]^2 \tag{2}$$

And q^{th} order of the flucuation function for a given value of q can be calculated following equation (3):

$$[F_v(s)]^q = \frac{1}{s}\sum_{j=1}^{s}\left[\in \left((v-1)s + j\right)\right]^q \tag{3}$$

Where q can take any real value except 0. Relationship between fluctuation function and s is governed by power law and can be described by equation (4):

$$F_q(s) \sim s^{h(q)} \tag{4}$$

These fractals are explained by singularity spectrum $f(\alpha)$ given by equation (5):

$$f(\alpha) = q \cdot \alpha - \tau(q) \tag{5}$$

Where $f(\alpha)$ explains the fractal dimension of the multifractals, α represents singularity strength, q represents the slope of the spectrum while $\tau(q)$can be expressed by generalized Hurst exponent $H(q)$ in equation (6):

$$H(q) = \lim_{q' \to q}\left(\frac{\tau(q')+1}{q'}\right) \tag{6}$$

In a multivariate case of two series (Jiang et al., 2019; Podobnik & Stanley, 2008; Zhou, 2008) with equal number of observations $\{X(i)\}$ and $\{Y(i)\}$ where $i = 1, \cdots, N$. each time series is divided into segments of equal size s such that they do not overlap, as it is a case when univatiate series is considered. The cross-correlation in each segment is given in equation (7):

$$F_v^2(s) = \frac{1}{s}\sum_{k=1}^{s}\left[X_v(k) - \widehat{X_v(k)}\right]\left[Y_v(k) - \widehat{Y_v(k)}\right] \quad (7)$$

Cross-correlation between two series or the q^{th} order is provided in equation (8):

$$F_{xy}^2(q,s) = \left[\frac{1}{m}\sum_{v=1}^{m} F_v(s)^q\right]^{\frac{1}{q}} \quad (8)$$

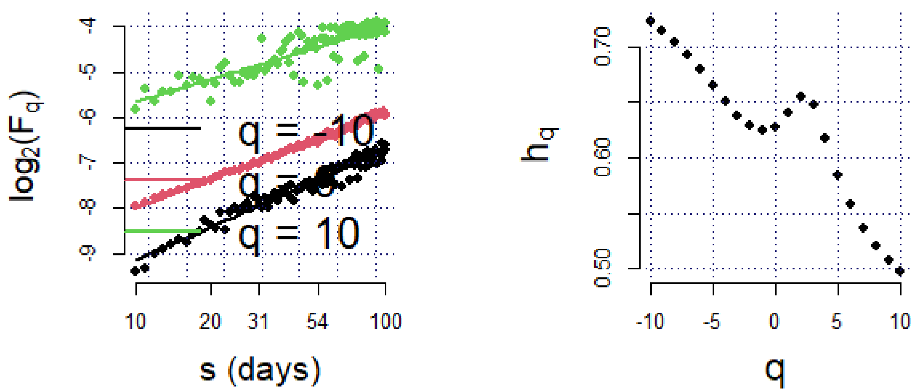
While scaling relation is given in equation (9):

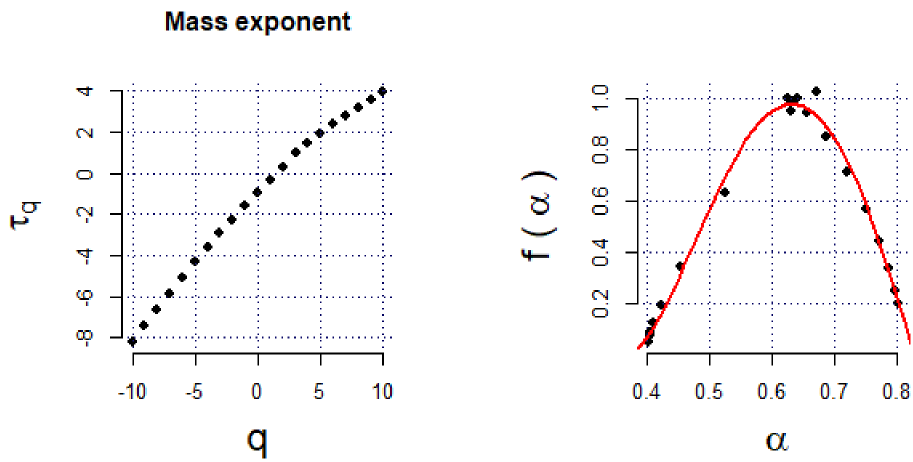
$$F_{xy}(q,s) \sim sH_{xy}(q) \quad (9)$$

RESEARCH RESULTS AND DISCUSSION

Research data used in this paper were introduced and described in appendix of this paper. Figure A1 and A2 present development of the time series while Table 1 and 2 provided descriptive statistics for the time series under consideration. Following methods described in section entitled methodology, Figure 1 presents fluctuation function, scaling exponent, multifractal exponent and singularity spectrum for CROBEX returns.

Figure 1. CROBEX returns

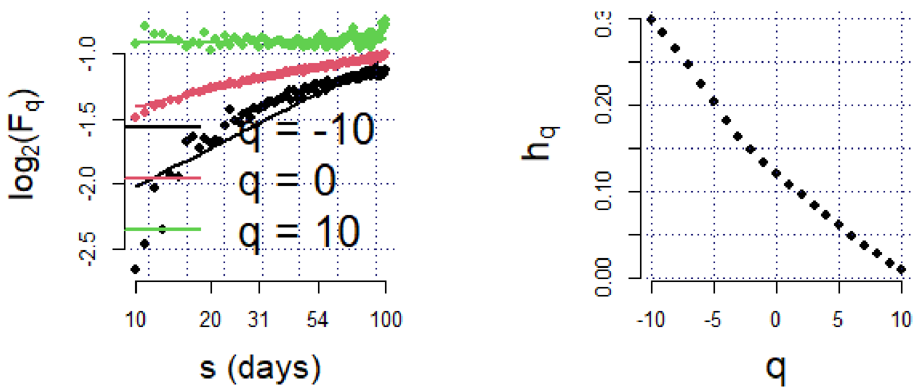


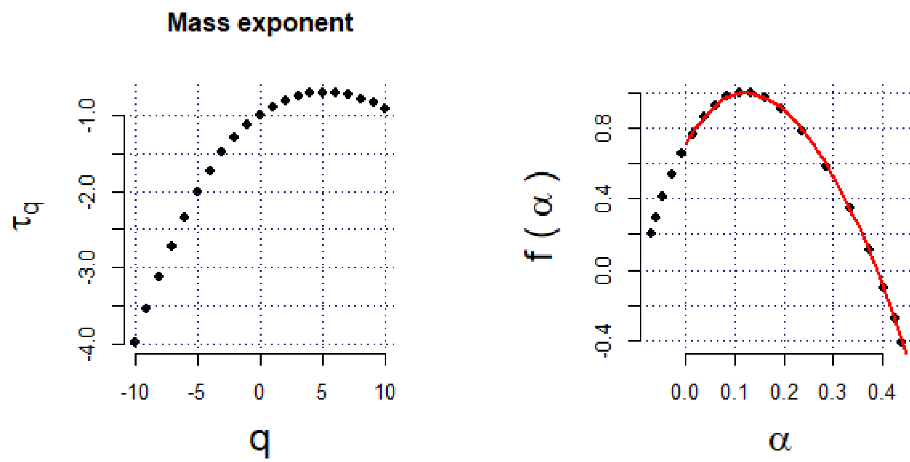


Source: Prepared by the authors

As illustrated in Figure 1, fluctuation function increases for large values of (days) demostrating existence of long-range power law correlation in the series as given in equation (4). The plot of versus illustrated decreasing trend and suggesting multifractality of the time seires. Since Hurst exponent takes values between 0.5 and 1 for positive and negative values (Kantelhardt et al., 2002) the series exhibits presence of long memory for large fluctuations as well as for small fluctuations. Furthermore, the relationship between multifractal exponents and is nonlinear that suggests multifractality of the time seires as well. The spectrum of the series is right-shifted from suggesting persistence of long memory in series. Figure 2 presents fluctuation function, scaling exponent, multifractal exponent and singularity spectrum for CROBEX trading volumes.

Figure 2. CROBEX trading volumes

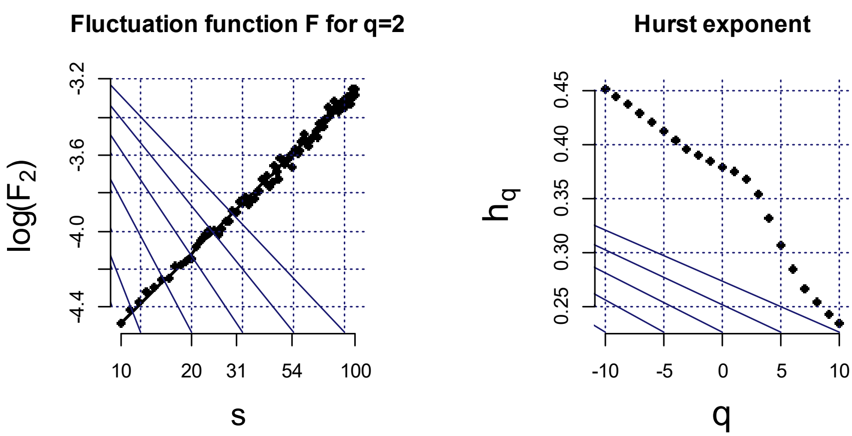


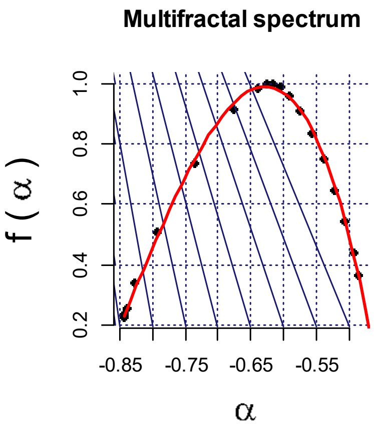


Source: Prepared by the authors

Unlike dynamics in Figure 1, no persistence was found in dynamics of CROBEX trading volume. Mass exponent suggested multifractality of time series as well as in case of CROBEX returns. The spectrum of the series is left-shifted from suggesting no long memory or no persistence in series. Figure 3 presents price-volume relationship in case of CROBEX.

Figure 3. price-volume relationship in case of CROBEX

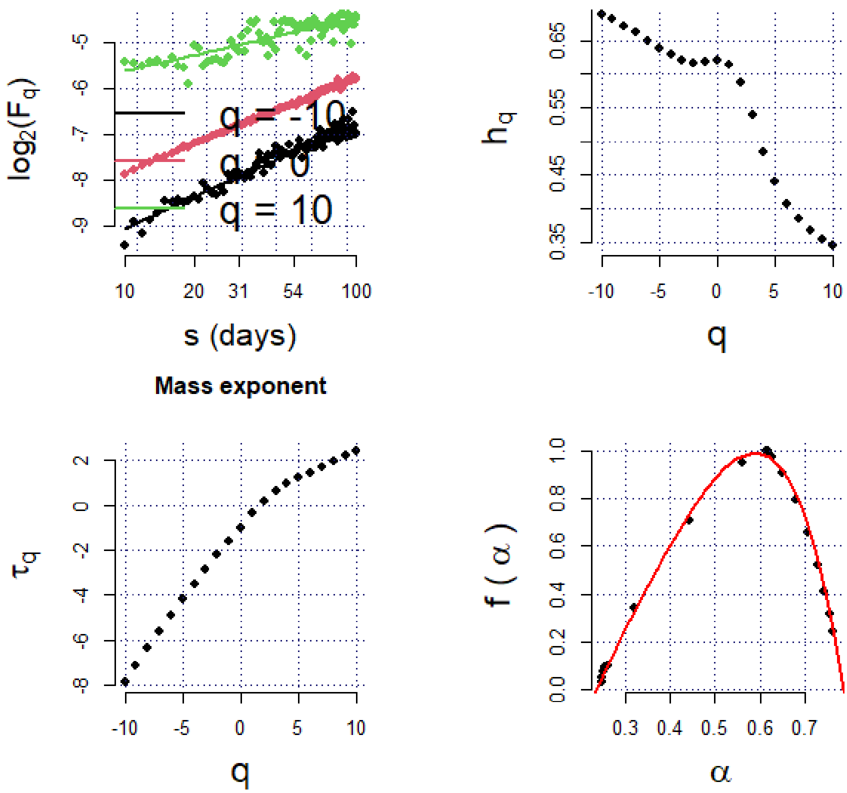




Source: Prepared by the authors

Results in Figure3 suggested that price-volume cross-correlation had no long memory. Consequently, volume and price-volume were found as mean-reverting series while price series exhibited long memory property in case of CROBEX. Bošnjak (2023) found higher liquidity linked to market inefficiency in CROBEX returns. Figure 4 presents properties of BELEX returns dynamics.

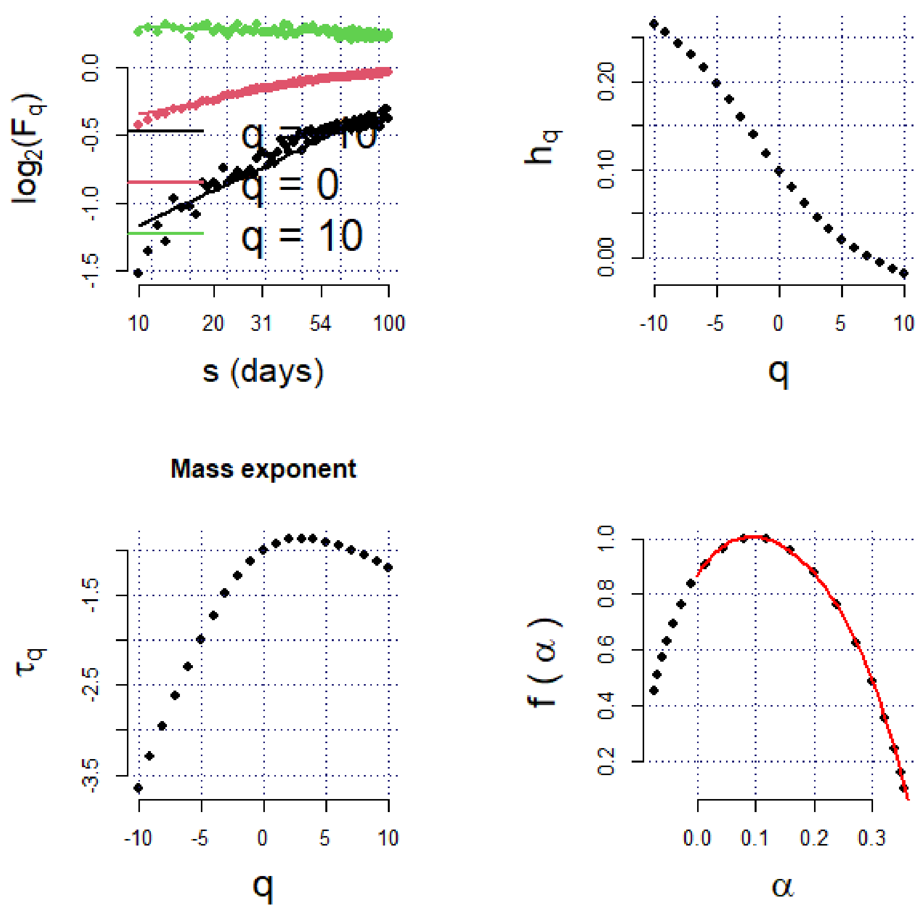
Figure 4. BELEX returns



Source: Prepared by the authors

Fluctuation function increases for large values of (days) demostrating existence of long-range power law correlation in the series as given in equation (4). The plot of versus illustrated decreasing trend and suggesting multifractality of the time series. Since Hurst exponent takes values higher than 0.5 for negative values small fluctuations seem to be persistent while large fluctuations appeared as with no long memory mostly. The series was further characterized by multifractality and the spectrum of the series is slightly right-shifted from suggesting long memory or persistence in series. Figure 5 illustrates dynamics of BELEX trading volume.

Figure 5. BELEX trading volumes



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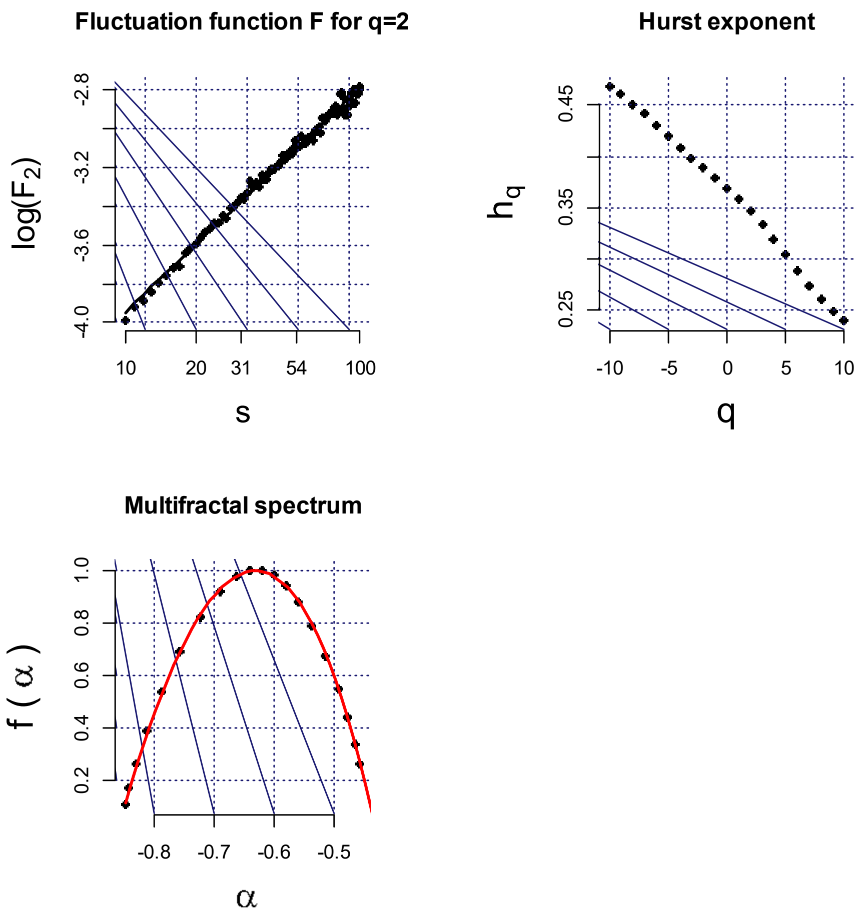
As well as in case of CROBEX trading volumes, BELEX trading volumes appeared left-shifted from suggesting no long memory or no persistence in series. Mass exponent suggested multifractality of time series as well as. Figure 6 price-volume relationship in case of BELEX.

Figure 6. price-volume relationship in case of BELEX



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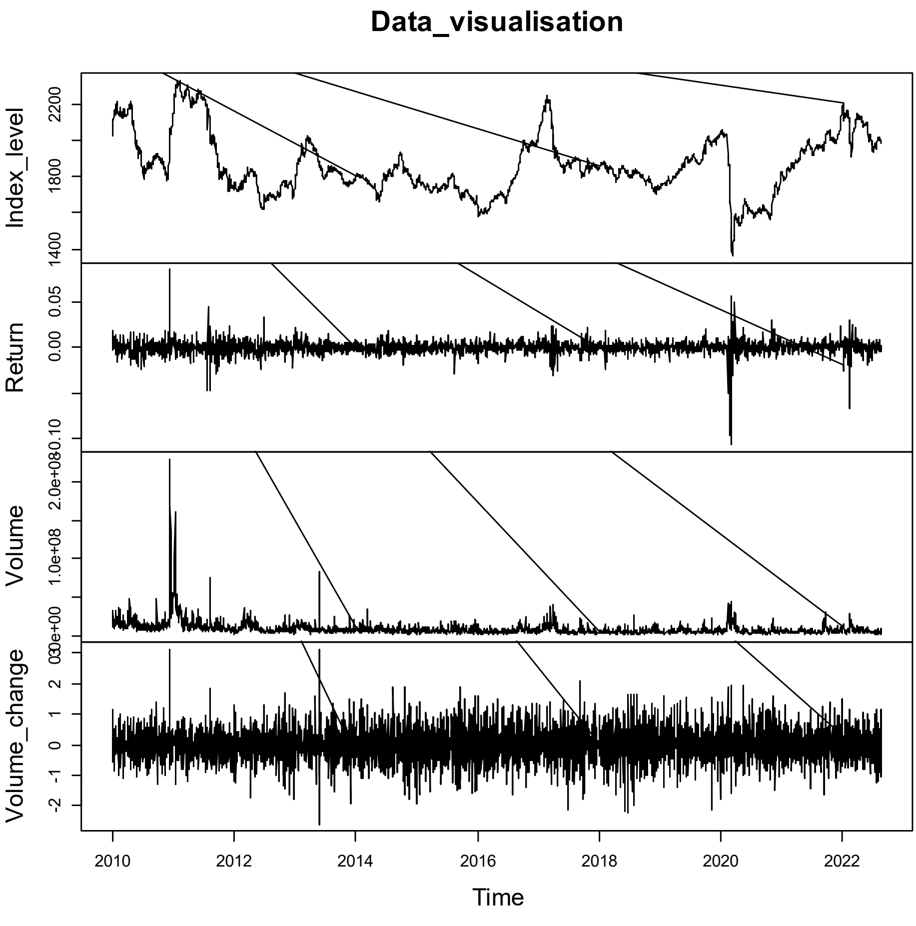
Results in Figure 6 suggested that price-volume cross-correlation had no long memory. Hence, there was multifractality and long memory properties of returns on both markets considered in this paper. No long memory was found in trading volumes neither in price-volume cross correlation.

CONCLUDING REMARKS

There are several conclusions that can be derived out of the research presented in this paper. Firstly, existing empirical literature suggest ambiguous findings regarding long memory property of stock markets. Empirical finings suggested multifractality of CROBEX returns and trading volumes and presence of long memory for large fluctuations in CROBEX returns as well as for small fluctuations in CROBEX returns. No long memory or no persistence was found in trading volumes on Croatian stock market. Price-volume cross-correlation on Croatian stock market was not characterized by long memory. Consequently, volume and price-volume were found as mean-reverting series while price series exhibited long memory property in case of CROBEX. Empirical finings for BELEX15 returns suggested multifractality of the time series. Small fluctuations seem to be persistent while large fluctuations appeared mostly with no long memory. Trading volumes on Serbian stock market exhibited no long memory or no persistence and multifractality of trading volumes. Price-volume cross-correlation exhibited no long memory property.

APPENDIX

Figure A1. Considered series in Croatian Case



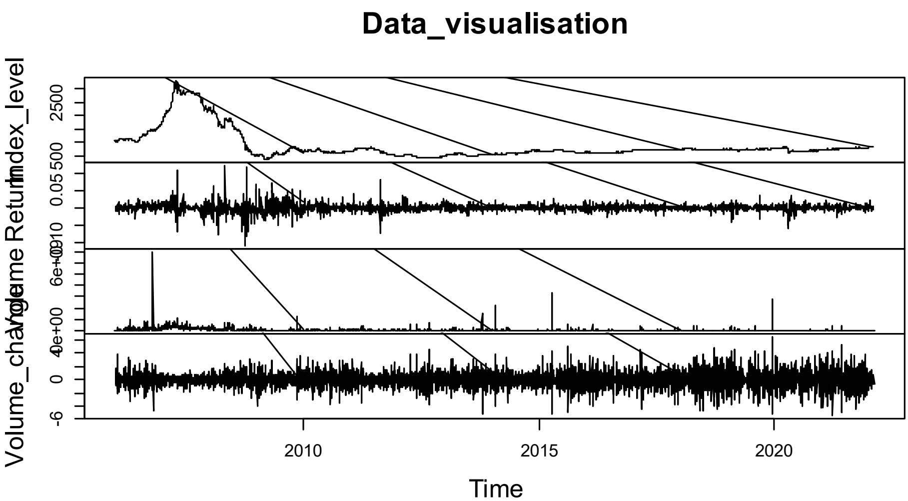
Source: Prepared by the authors

Table 1. Descriptive statistics for CROBEX time series

	Index	Index change	Volume	Volume change
Min.	1365	-0.1073230	757625	-2,6140175
1st Qu.	1752	-0.0031919	4042911	-0.3736423
Median	1837	0.0001860	6192440	-0.0157923
Mean	1862	-0.0000062	8576433	-0.0005984
3rd Qu.	1962	0.0035157	9614106	0.3689675
Max.	2334	0.0856288	230466363	3,1403988

Source: Author's calculations

Figure A2. Considered series in Serbian Case



Source: Prepared by the authors

Table 2. Descriptive statistics for BELEX15 time series

	Index	Index change	Volume	Volume change
Min.	354.4	-0.1086135	26130	-5,579589
1st Qu.	608.4	-0.0041540	8032000	-0.678174
Median	706.6	-0.0000216	19760000	0.014850
Mean	863.5	-0.0000616	58770000	-0.000649
3rd Qu.	770.8	0.0042101	49010000	0.662204
Max.	3304.6	0.1215756	6987000000	6,490316

Source: Author's calculations

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